

A Deduction System for Meaning Negotiation

Elisa Burato, Matteo Cristani, and Luca Viganò

Dipartimento di Informatica, Università di Verona,
Cà Vignal 2, strada Le Grazie 15, I-37134 Verona
e-mail: {elisa.burato,matteo.cristani,luca.vigano}@univr.it

Abstract. Meaning negotiation (MN) is the general process with which agents reach an agreement about the meaning of a set of terms. We give here a general model of MN for two agents, in which each agent discusses with the other one her viewpoint by exhibiting it in an actual set of constraints on the meaning of the negotiated terms. We call this presentation of individual viewpoints an angle. The two agents do not aim at forming a common viewpoint but, instead, at agreeing about an acceptable common angle. We formalize the process of reaching such an agreement by giving a deduction system that comprises of rules that are consistent and adequate for representing MN.

1 Introduction: Context and Contributions

A *negotiation* is a dialog (i.e., a conversation between two or more agents) intended to resolve disputes, to produce an agreement upon courses of action, to bargain for individual or collective advantage, or to craft outcomes to satisfy various interests. *Meaning negotiation* (MN) is the process that takes place when the involved agents have some knowledge (some data or information) to share but do not agree on what knowledge agents share and how they reach an agreement about it. The knowledge of an agent represents her viewpoint and we call *angle* any partial representation of the viewpoint; therefore, the knowledge of a negotiating agent is built by a single viewpoint and many angles. In this paper, we assume that angles are presented as *logical theories*, and in particular *propositional* ones. At the beginning of a MN process, agents are in disagreement, i.e., they have mutually inconsistent knowledge. By MN, they try to reach a common angle representing a shared acceptable knowledge, where the MN ends in positive way when the agents have a common knowledge, and it ends in a negative way otherwise: agents are in

- *agreement* when they have the same knowledge, namely they have found a set of constraints on the meaning of the negotiated terms that is accepted by both agent (this new theory is named, here, a *common angle*);
- *disagreement* when they are not in agreement.

Negotiation of meaning has been considered, directly or indirectly, in a large number of works, ranging from works focusing on ontology (see [7, 9]) to works dealing with contexts (e.g., [6]) or, more generally, to research in the field of

knowledge representation (e.g., [4]). Only a few works, however, have considered MN as a process; for instance, [11, 5] deal with MN as an ontology alignment process, and [13, 8, 1, 14, 2] deal with negotiation issues from the point of view of game theory. In particular, a MN between two agents is similar to a *Bargaining Game* [10], i.e., the game in which two agents have to share, say, one dollar and do this by each making a proposal. If the sum of their demands is less than one, they share the dollar, otherwise they have to make a new demand. The Bargaining Game is built by two stages:

- *Demand stage*: agents make a proposal and if the proposals are compatible, the negotiation ends in positive way; otherwise the second stage begins.
- *War of attrition*: agents have incompatible viewpoints and perform new demands. If the demands are compatible, the process ends positively, otherwise they make new ones.

In the Bargaining Game, players have a *negotiation power* that represents how often an agent cedes during the negotiation and how much she resists about her current angle. The negotiation power of an agent is captured by a set of partially ordered angles of her viewpoint. The partial order among the angles allows an agent to choose the next proposal to perform, and to evaluate the acceptability of the received offers. Moreover, the set of partially ordered angles has a minimum that identifies the last offer an agent proposes in a negotiation. We say that each agent has a *stubborn* and many *flexible*¹ angles that are respectively the limit proposal (i.e., the last offer) and the acceptable ones.

Example 1. To define the term “vehicle”, Alice thinks that it always has two, three, four or six wheels, and a handlebar or a steering wheel, and a motor or two or four pedals. However, Alice thinks that a “vehicle” may be defined only as a car, then having four wheels, a steering wheel and a motor; otherwise only as a bike, then having two wheels, a handle bar and two pedals.

In the example above, Alice has two acceptable ways to define a vehicle (a car or a bike as particular “vehicles”) but she has only one general description of a “vehicle”.

The MN stages, shown in Figure 1(a), are the following ones:

- *Init*: the first bidding agent makes a proposal;
- *Negotiate*: the agents propose their viewpoints in turns and evaluate whether they agree with the opponents;
- *Agreement*: all² the agents agree on a common viewpoint;
- *Disagreement*: the agents do not have a shared viewpoint.

The negotiation power of an agent is known only by herself, and when an agent makes a proposal, she includes stubborn and flexible knowledge in it. Conversely, when looking at the knowledge of the other agent, one is not able to say on what knowledge the other agent would be stubborn.

¹ Each flexible angle is consistent with the stubborn knowledge.

² [3] consider more than two negotiating agents and formalize a partial positive outcome, in which a degree of sharing denotes the minimum number of agreeing agents needed to consider the MN as positive.

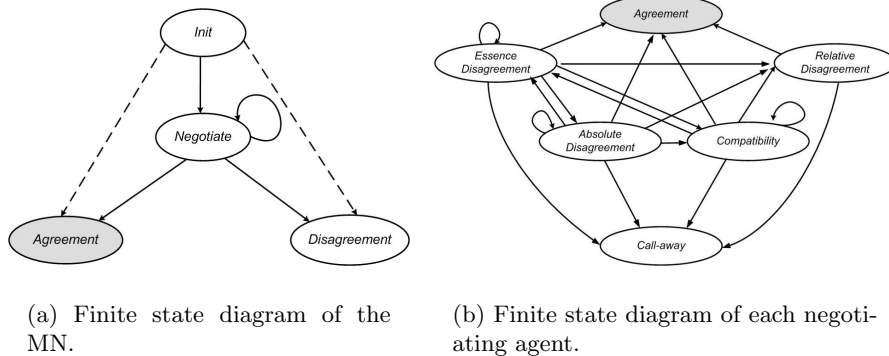


Fig. 1. Finite state diagrams of: the negotiation process (a), and of the negotiating agent (b).

The *first contribution* of this paper is the definition of a general model of MN in which the two agents have different viewpoints that are not completely compatible. In our model, the types of disagreement depend upon the relation among the proposal p and the stubborn and the flexible knowledge of the agent i who receives and evaluates p :

- *Call-away* occurs when p is a generalization³ of the stubborn knowledge of i , thus it would correspond to drop out some unquestionable knowledge.
- *Absolute disagreement* occurs when the stubborn knowledge of i is inconsistent with respect to p .
- *Essence disagreement* occurs when the flexible knowledge of i is inconsistent with respect to p .
- *Compatibility* occurs when p is consistent with the flexible knowledge of i but it is not a generalization or a restriction of i 's viewpoint.
- *Relative disagreement* occurs when p is a generalization of the flexible knowledge of i .

The call-away situations arise when an agent does not accept all the necessary requests of the other one and thus exits the MN and the MN negatively ends. In Figure 1(b), we show the finite state diagram for each negotiating agent, where the disagreement node of Figure 1(a) is expanded to the types of disagreement we consider here. In a MN process, all the states can be initial and final and the *agreement* state is the optimal final one.

The *second contribution* of this paper is the formalization of a deduction system, which we call *MND*, to reason about the MN process. The two agents start the negotiation process with an initial proposal and concede to each other about the other's viewpoint until a common definition of the terms is obtained.

³ A theory A is a *generalization* of a theory B when the models of A are a superset of the models of B .

Each of the two agents has a limit in negotiation, since some of her knowledge is unquestionable and, therefore, she will never concede about it. Consequently, after being flexible for a first phase of the negotiation process, when the agreement cannot be obtained, the agent becomes stubborn about her unquestionable knowledge. If this situation is symmetric, the disagreement condition becomes perpetual and the two agents keep on proposing the same incompatible definitions for the terms under negotiation. The system controls the procedure in what condition is reached. When the agreement condition is reached, the two agents agree about a common definition of the terms and the system ends the negotiation with positive outcome; when the agents reach a perpetual disagreement condition, the system ends the negotiation by stating that the agreement cannot be reached.

MND allows us to express that agents communicate to each other not only the proposals, but also the disagreement conditions they have reached so far. The process is governed by a set of rules that manage the provisional disagreement condition the agents have reached. We first provide rules for deriving streams of dialog between two agents who discuss about the meaning of a set of terms, and then define a deduction system based upon these rules that derives a stream of dialog that ends with an agreement/disagreement condition.

We show that *MND* is consistent and adequate to represent the MN of two agents. Moreover, MN is decidable over theories with finite signature under the assumption of agents who are competitive (in a sense to be defined precisely below).

The paper comprises of two further sections. We first formalize the knowledge and the language of negotiating agents, and the language and rules of the MN process. Then, we draw conclusions and discuss future work.

2 A Formalization of Negotiating Agents

We consider here a general MN process, so we abstract away from the particular terms whose meaning the agents are negotiating. We first consider the knowledge (§ 2.1) and language (§ 2.2) of negotiating agents, and then formalize the MN process (§ 2.3), language (§ 2.4) and rules (§ 2.5).

2.1 The Knowledge of Negotiating Agents

When agents give the definition of a concept, they: give the necessary (properties about which the agent is stubborn - in short *stubborn properties*) properties and the characterizing (properties about which the agent is flexible - in short *flexible properties*) ones; give the properties that necessarily have not to hold and the ones that plausibly (flexibly) have not to hold; and give the formulas asserting what has not to (stubbornly), or may not (flexibly), be used in the definition.

The notion of relevance of a formula is interesting at this stage of the definition, but instead of introducing a novel operator, we simply consider a formula as not relevant to an agent if she does not assert it. When *i asserts* a formula

φ , she has a way to evaluate it: she thinks φ as positive or negative. If i does not assert φ then either i does not know φ , i.e., she is not able to evaluate it or i does not think φ is relevant in defining the negotiated meaning.

The necessary and the characterizing properties of a concept definition are closely related to *EGG/YOLK* objects, introduced by [12] to represent class membership based on typicality of the members: the egg is the set of the class members and the yolk is the set of the *typical* ones. For instance, the class of “employees” of a company A may be defined as “the set of people that receive money from the company in exchange for carrying out the instructions of a person who is an employee of that company”, thus excluding, e.g., the head of the company (who has no boss), and the typical employee would include regular workers like secretaries and foremen. Another company B might have a different definition, e.g., including the head of the company, resulting in a mismatch. Nevertheless, if both companies provide some typical examples of “employees” it is possible that all of A ’s typical employees fit B ’s definition, and all of B ’s typical employees fit A ’s definition: $YOLK_B \leq EGG_A$ and $YOLK_A \leq EGG_B$, in the terminology of [12].

Differently than in the original model, concept definitions are here restricted by stubborn properties to the largest acceptable set of models, hence represented by the egg, whilst the yolk is employed to denote the most restricted knowledge, that is, the one on which the agents are flexible and they may cede about it.

The stubborn properties never change during the negotiation; therefore, the egg is fixed at the beginning of the MN. Instead, the flexible part of the definition of a concept is the core of the proposal of a negotiating agent. Each proposal differs from the further ones in two possible ways: it may give a definition of the negotiated object that is more descriptive than the next ones, or the given definition specifies properties that the next ones do not and vice versa. In the former case, we say that the agent carries out a *weakening action*, in the latter the agent carries out a *changing theory action*.

However, none of weakening or changing theory actions can be carried out with respect to a proposal if the proposal describes the necessary properties of the object in the MN. We say that in such a situation the agents always make a *stubbornness action* that is equivalent to *no more change*.

2.2 The Language of Negotiating Agents

Each agent i is represented by her language \mathcal{L}_i , which is composed of two disjoint sublanguages: a *stubbornness* language containing the properties i deems as necessary in defining the negotiated meaning and a *flexible* language containing the properties i deems as not necessary in the MN.

Definition 1. *Let Ag be the set of the negotiating agents. The signature Σ_i of an agent $i \in \text{Ag}$ is the pair $\langle \mathcal{P}_i, \alpha_i \rangle$ where \mathcal{P}_i is the set of the predicate symbols and α_i is the arity function for predicate symbols $\alpha_i : \mathcal{P}_i \rightarrow \mathbb{N}$.*

The language \mathcal{L}_i of $i \in \text{Ag}$ comprises of Σ_i -formulas defined as follows: (i) if $P \in \mathcal{P}_i$, $\alpha_i(P) = n$ and t_1, \dots, t_n are terms then $P(t_1, \dots, t_n)$ is a Σ_i -formula; (ii) if φ and ψ are Σ_i -formulas then so are $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, and $\varphi \rightarrow \psi$.




$\frac{flex_i^k \rightarrow flex_i^{k+1} \quad \neg(stub_i \leftrightarrow flex_i^k)}{flex_i^{k+1}} \quad (W)$	
$\frac{flex_i^k \quad \neg(stub_i \leftrightarrow flex_i^k) \quad \neg(flex_i^k \rightarrow flex_i^{k+1})}{flex_i^{k+1}} \quad (C)$	
$\frac{\varphi \quad stub_i \leftrightarrow \varphi}{\varphi} \quad (S)$	

Table 1. Rules for making new proposals and the corresponding EGG/YOLKS. The dashed ellipse identifies $flex_i^{k+1}$ and the plain gray filled one identifies $flex_i^k$.

$\mathcal{L}_i = \mathcal{L}_{S_i} \cup \mathcal{L}_{\mathcal{F}_i}$ where the set \mathcal{L}_{S_i} of stubborn formulas is disjoint from the set $\mathcal{L}_{\mathcal{F}_i}$ of flexible formulas. We define $stub_i = \bigwedge_{\varphi \in \mathcal{L}_{S_i}} \varphi$ and $flex_i = \bigwedge_{\varphi \in \mathcal{L}_{\mathcal{F}_i}} \varphi$.

During a negotiation process, the viewpoint of each agent is presented in a specific *angle*. In other words, a viewpoint is a hierarchy of theories, related by the partial order relation of weakening, and an element of this hierarchy is an angle. Each agent presents angles in sequence during the negotiation. Thus we call *current angle formula (CAF)* the angle presented at the current stage of the negotiation. A flexible formula $flex_i^k$ expresses the k^{th} angle discussed in the MN and it changes during the process. We assume here that for each CAF $flex_i^k$ there is a stubborn formula in \mathcal{L}_{S_i} that is a generalization of it. In general, during a negotiation of the meaning of a term, the agents relax their viewpoint in order to meet the opponent's one, and they do this only if the relaxing formula is not too general. Then, for each assertion in the MN, the agents have a maximal generalization of it and this is a formula in the stubbornness set. For instance, if the object of the negotiation is the meaning of *pen*, an agent is flexible on the ink color of the object but not on the fact that the object contains ink; then, the *red ink* predicate is a flexible one and the *contains ink* predicate is a stubborn one.

$flex_i^k$ changes during the MN by applying to it one of the rules for making new proposals given in Table 1: weakening (*W*), changing theory (*C*) or stubbornness (*S*). The EGG/YOLK representations show with dashed lines the collocation of the new proposal (in the stubbornness situation the new proposal is the same as the last one).

There are two ways for i to make a new proposal $flex_i^{k+1}$. The weakening rule (*W*) states that i can propose $flex_i^{k+1}$ if $flex_i^{k+1}$ is entailed by $flex_i^k$ (i.e., $flex_i^k \rightarrow flex_i^{k+1}$) and $flex_i^k$ is not the most general formula the agent can negotiate (corresponding to her stubbornness viewpoint, i.e., $flex_i^k \leftrightarrow stub_i$). Note that if i weakens, say, $flex_i^0$ to the new CAF $flex_i^1$, then i may be no more able to satisfy $flex_i^0$.

The rule (*C*) states that i can just change theory. Although we do not consider MN strategies in detail here, in general, an agent chooses whether to perform a weakening or a changing theory action by applying the corresponding rule, but

there are situations in which one action is better than the other. For instance, when an agent checks the compatibility situation it seems better to weaken the theory so to try to entail the opponent's viewpoint, while in essence disagreement situations it seems better to change the theory so to try to meet the opponent's viewpoint.

If agent i is in stubbornness does she continue the negotiation or does she have to exit it? We assume that the agent exits the MN only if all the agents in the negotiation are stubborn. But an agent does not know the opponent's stubbornness viewpoint, so the exit condition is recognized only by the system. However, the stubborn agent always makes the same proposal during the MN, as expressed by the rule (S). If $flex_i^k \leftrightarrow stub_i$ then $flex_i^{k_1} = flex_i^{k_1+1}$ for all $k_1 > k$.

We introduce a set of Σ_i -structures as agents change angles during the negotiation process and these viewpoints have to be satisfied in a different structure. We use a parameter k to denote the k^{th} structure of the k^{th} angle.

Definition 2. Given a signature $\Sigma_i = \langle \mathcal{P}_i, \alpha_i \rangle$, a Σ_i -structure \mathcal{A}_i is a pair $\langle \mathcal{D}_i, \mathcal{I}_i \rangle$ where the domain \mathcal{D}_i is a finite non-empty set and the interpretation function \mathcal{I}_i is such that $\mathcal{I}_i(P) \subseteq \mathcal{D}_i^n$ for all $P \in \mathcal{P}_i$ for which $\alpha(P) = n$.

We define $\mathcal{S}_i = \{\mathcal{A}_i^k \mid \mathcal{A}_i^k = \langle \mathcal{D}_i^k, \mathcal{I}_i^k \rangle\}$ where $\mathcal{D}_i^k \subseteq \mathcal{D}_i$ and, for all $(\mathcal{I}_i^k, \mathcal{I}_i^{k+1})$, if the $(k+1)^{\text{th}}$ rule that i applied is

- (W), then $\mathcal{I}_i^k(P) \subseteq \mathcal{I}_i^{k+1}(P)$ for all $P \in \mathcal{P}_i$;
- (C), then $\mathcal{I}_i^k(P) \neq \mathcal{I}_i^{k+1}(P)$ and $\mathcal{I}_i^k(P) \not\subseteq \mathcal{I}_i^{k+1}(P)$, for all $P \in \mathcal{P}_i$;
- (S), then $\mathcal{I}_i^k(P) = \mathcal{I}_i^{k+1}(P)$, for all $P \in \mathcal{P}_i$.

If φ and ψ are Σ_i -formulas then:

- $\mathcal{A}_i^k \models P(t_1, \dots, t_n)$ iff $(\mathcal{I}_i(t_1), \dots, \mathcal{I}_i(t_n)) \in \mathcal{I}_i(P)$, where $P \in \mathcal{P}_i$ and t_1, \dots, t_n are terms;
- $\mathcal{A}_i^k \models \neg\varphi$ iff $\mathcal{A}_i^k \not\models \varphi$;
- $\mathcal{A}_i^k \models \varphi \wedge \psi$ iff $\mathcal{A}_i^k \models \varphi$ and $\mathcal{A}_i^k \models \psi$;
- $\mathcal{A}_i^k \models \varphi \vee \psi$ iff $\mathcal{A}_i^k \models \varphi$ or $\mathcal{A}_i^k \models \psi$;
- $\mathcal{A}_i^k \models \varphi \rightarrow \psi$ iff $\mathcal{A}_i^k \models \varphi$ or $\mathcal{A}_i^k \not\models \psi$.

Example 2. Suppose Alice defines “vehicle” as in Example 1. Then

$$\begin{aligned} stubA = & (\text{has2wheels} \vee \text{has3wheels} \vee \text{has4wheels} \vee \text{has6wheels}) \wedge \\ & (\text{hasHandleBar} \vee \text{hasSteeringWheel}) \wedge \\ & (\text{hasMotor} \vee \text{has2pedals} \vee \text{has4pedals} \vee \text{isDrawn}) \end{aligned}$$

is the stubbornness part of Alice's knowledge whose interpretation is $\mathcal{I}(stubA) = \{\text{bike, tandem, motorbike, scooter, truck, car, trailer, chariot}\}$. Let

$$flex_A^k = \text{has4wheels} \wedge \text{hasSteeringWheel} \wedge \text{hasMotor}$$

the CAF of Alice that it is not equivalent to her stubbornness knowledge and its interpretation is $\mathcal{I}(flex_A^k) = \{\text{car, truck}\} \subset \mathcal{I}(stubA)$. Suppose Alice changes her CAF by means of a weakening action (W), then:

$$flex_A^{k+1} = (\text{has4wheels} \vee \text{has2wheels}) \wedge (\text{hasSteeringWheel} \vee \text{hasHandleBar}) \wedge \text{hasMotor}$$

The interpretation of $flex_A^{k+1}$ is $\mathcal{I}(flex_A^{k+1}) = \{\text{motorbike, scooter, car, truck}\} \subset \mathcal{I}(flex_A^k)$.

Otherwise suppose Alice changes her CAF by means of a changing theory action (C), then:

$$flex_A^{k+1} = \text{has6wheels} \wedge (\text{hasSteeringWheel} \wedge (\text{hasMotor} \vee \text{isDrawn}))$$

The interpretation of $flex_A^{k+1}$ is $\mathcal{I}(flex_A^{k+1}) = \{\text{truck, trailer}\}$ and $\mathcal{I}(flex_A^{k+1}) \not\subset \mathcal{I}(flex_A^k)$.

2.3 The MN Process

We now formalize the MN process for two agents. During the MN, agents make proposals and say if they are in agreement or not with respect to the proposals made by the opponent. Proposals are negotiation formulas like $j : \varphi$, where we assume that the opponent i is able to recognize the name label j in $j : \varphi$ and remove it in order to evaluate φ .

In general, negotiating agents may not share the same language but have different signatures. Hence, when i evaluates an assertion by j , she first has to *translate* the symbols occurring in it to symbols belonging to her signature. Such a translation depends, of course, on the particular terms that are being considered for the negotiation, so we assume abstractly that for each pair of agents (i, j) there is the *translation function*

$$\tau_{i,j} : \Sigma_j \rightarrow \Sigma_i.$$

When j asserts φ (i.e., $j : \varphi$), i is not able to find which part of φ is in the stubbornness set of j , since she only knows that $\varphi = \text{stub}_j \wedge \psi^k$ where stub_j is the conjunction of all the formulas in \mathcal{L}_{S_j} and ψ^k is the k^{th} angle of j .

In the following, we describe the main conditions an agent has to test in order to evaluate the opponent proposal and to identify the negotiation condition she is in. We suppose that j is the first proponent (bidding) agent and that i is the agent evaluating j 's proposal. Table 2 shows the EGG/YOLK representations in which i is identified by the plain line and j by the dashed line for each condition i tests; the numbering is that of [12]. Let φ be the proposal of j ; then, the main conditions i has to test are (where, as usual, consistency means the impossibility to derive \perp):

- $(\text{stub}_i \rightarrow \tau_{i,j}(\varphi))$: are the agents in a *call-away* situation, i.e., is the proposal of j a generalization of the stubbornness set of i ? If it is the case, then the MN process ends negatively. The corresponding EGG/YOLK representation is shown in Table 2(a).
- $\neg(\text{stub}_i \wedge \tau_{i,j}(\varphi))$: is the proposal of j consistent with respect to i 's stubbornness set? If it is not, then the agents are in absolute disagreement (Table 2(b)).
- $\neg(\text{flex}_i^k \wedge \tau_{i,j}(\varphi)) \wedge (\text{stub}_i \vee \tau_{i,j}(\varphi))$: i and j are not in absolute disagreement; is i 's CAF consistent with respect to j 's proposal? If it is not, then the agents are in essence disagreement (Table 2(c)).

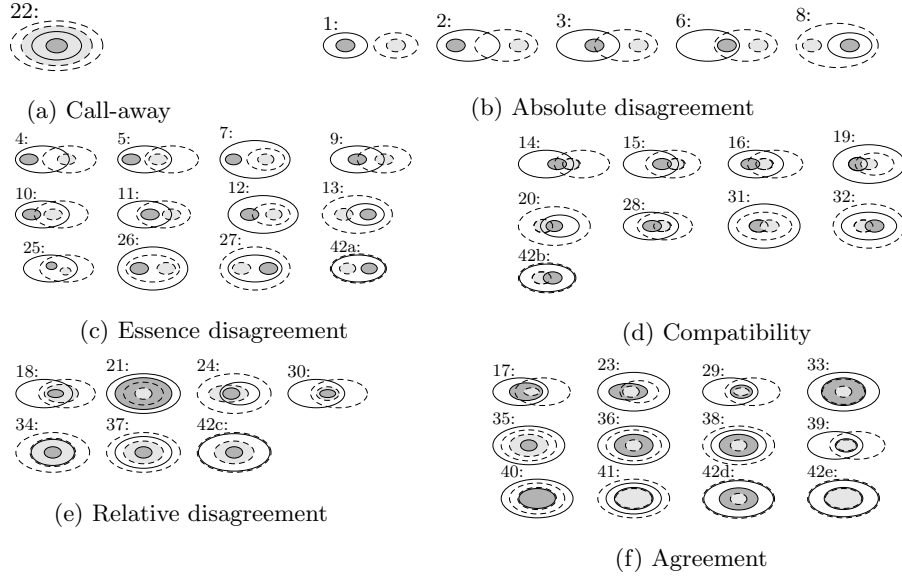


Table 2. EGG/YOLK representation of the opponent's offer from agent i 's viewpoint.

- $(flex_i^k \rightarrow \tau_{i,j}(\varphi)) \wedge \neg(\tau_{i,j}(\varphi) \rightarrow flex_i^k)$: i and j are not in essence nor in absolute disagreement; is j 's proposal a generalization of i 's CAF? If it is and if i 's CAF is not equivalent to j 's proposal, then the agents are in relative disagreement (Table 2(e)).
- $(flex_i^k \vee \tau_{i,j}(\varphi)) \wedge \neg(flex_i^k \rightarrow \tau_{i,j}(\varphi)) \wedge \neg(\tau_{i,j}(\varphi) \rightarrow flex_i^k)$: i and j are not in absolute nor in relative disagreement; is i 's CAF consistent with respect to j 's proposal? If it is and if i 's CAF is not a weakening of j 's proposal, then the agents are in the compatibility relation (Table 2(d)).
- $(flex_i^k \leftrightarrow \tau_{i,j}(\varphi))$: the proposal of j is equivalent to i 's CAF. The agents are in agreement (Table 2(f)).

After evaluating the received proposal, agents inform the opponent about the negotiation situation they think to be in. To this end, we extend the formulas in the agent language:

Definition 3. If φ is a received proposal in the negotiation process, then it is a formula asserted by somebody as $j : \varphi$. We extend the language \mathcal{L}_i with the formulas **absDis**($j : \varphi$), **essDis**($j : \varphi$), **relDis**($j : \varphi$), **comp**($j : \varphi$), and **agree**($j : \varphi$). For $\mathcal{A}_i^k = \langle \mathcal{D}_i^k, \mathcal{I}_i^k \rangle$ a Σ_i -structure, the semantics of these additional formulas is:

- $\mathcal{A}_i^k \models \mathbf{absDis}(j : \varphi)$ iff $\mathcal{A}_i^k \models \neg(stub_i \wedge \tau_{i,j}(\varphi))$;
- $\mathcal{A}_i^k \models \mathbf{essDis}(j : \varphi)$ iff $\mathcal{A}_i^k \models (stub_i \vee \tau_{i,j}(\varphi)) \wedge \neg(flex_i^k \wedge \tau_{i,j}(\varphi))$;
- $\mathcal{A}_i^k \models \mathbf{relDis}(j : \varphi)$ iff $\mathcal{A}_i^k \models (flex_i^k \rightarrow \tau_{i,j}(\varphi)) \wedge \neg(\tau_{i,j}(\varphi) \rightarrow flex_i^k)$;

- $\mathcal{A}_i^k \models \mathbf{comp}(j : \varphi)$ iff $\mathcal{A}_i^k \models (flex_i^k \vee \tau_{i,j}(\varphi)) \wedge \neg(flex_i^k \rightarrow \tau_{i,j}(\varphi)) \wedge \neg(\tau_{i,j}(\varphi) \rightarrow flex_i^k)$;
- $\mathcal{A}_i^k \models \mathbf{agree}(j : \varphi)$ iff $\mathcal{A}_i^k \models (flex_i^k \leftrightarrow \tau_{i,j}(\varphi))$.

We did not define a sentence **callAway**($j : \varphi$) as the call-away condition interrupts the MN. It is also important to remark that in our system we restrict the evaluation of agent proposals to formulas in the basic agent language, so no assertion can be made by agents using extended (and nested) formulas like **agree**(**comp**($j : \varphi$)). This restriction avoids nested MN processes.

In the two following subsections, we define the negotiation language and the deductive rules for the MN process.

2.4 MN language

The negotiation language, \mathcal{L} , is built by the assertions of the agents during the negotiation, i.e., labeled formulas $i : \varphi$ meaning that agent $i \in \text{Ag}$ asserts the formula $\varphi \in \mathcal{L}_i$. That is, $i : \varphi$ represents a proposal the agent i makes in the negotiation and typically represents her CAF.

Definition 4. *The signature of the MN language \mathcal{L} is $\Sigma = \langle \mathcal{P}, \{\alpha_i\}_{i \in \text{Ag}} \rangle$ where $\mathcal{P} = \bigcup_{i \in \text{Ag}} \mathcal{P}_i$ and $\alpha_i : \mathcal{P}_i \rightarrow \mathbb{N}$ is the arity function for predicate symbols. Let φ be a \mathcal{L}_i formula for some $i \in \text{Ag}$; then \mathcal{L} comprises of Σ -formulas defined as follows:*

- $i : \varphi$ is a Σ -formula;
- if φ_1 and φ_2 are Σ -formulas then $\varphi_1 \cap \varphi_2$ is a Σ -formula.

Let $\mathcal{N}^k = (\{\mathcal{A}_i^{k_i}\}_{i \in \text{Ag}, k_i \in \mathbb{N}}, \mathcal{F})$ be a Σ -structure where $\{\mathcal{A}_i^{k_i}\}_{i \in \text{Ag}, k_i \in \mathbb{N}}$ is the domain set and \mathcal{F} is an evaluation function that maps name labels into Ag . Then:

- $\mathcal{N}^k \models i : \varphi$ iff $\mathcal{A}_{\mathcal{F}(i)}^{k'} \models \varphi$ where $k' = \lceil \frac{k}{2} \rceil$ because the two agents make assertions in turns;
- $\mathcal{N}^k \models \varphi_1 \cap \varphi_2$ iff $\mathcal{N}^k \models \varphi_1$ and $\mathcal{N}^k \models \varphi_2$.

2.5 MN Rules

We now give the transition rules the agents use to negotiate depending on the mutual negotiation position they test and on their flexibility; these rules are coupled with those in Table 1. There are different rules for the second proposing agent and the following ones. Consider the scenario in Figure 2(a): Alice (A) makes the proposal φ and Bob (B) evaluates it, where B 's reasoning is based upon two tests:

1. The relation between his CAF and φ . B 's CAF may be in agreement ($\varphi \leftrightarrow flex_B^k$) or not with φ and B recognizes it by testing the condition listed above.

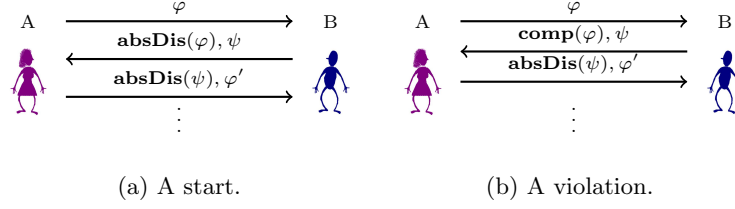


Fig. 2. Two MN scenarios.

$$\begin{array}{c}
\frac{j : \varphi \quad \neg(stub_i \wedge \tau_{i,j}(\varphi))}{i : \mathbf{absDis}(j : \varphi) \cap i : flex_i^1} \text{ (AD)} \quad \frac{j : \varphi \quad \neg(flex_i^0 \wedge \tau_{i,j}(\varphi)) \wedge (stub_i \vee \tau_{i,j}(\varphi))}{i : \mathbf{essDis}(j : \varphi) \cap i : flex_i^1} \text{ (ED)} \\
\frac{j : \varphi \quad (flex_i^0 \wedge \tau_{i,j}(\varphi)) \vee (\neg\tau_{i,j}(\varphi) \wedge flex_i^0)}{i : flex_i^0} \text{ (I)} \quad \frac{j : \varphi \quad (flex_i^0 \leftrightarrow \tau_{i,j}(\varphi)) \vee (flex_i^1 \leftrightarrow \tau_{i,j}(\varphi))}{i : \mathbf{agree}(j : \varphi) \cap i : \tau_{i,j}(\varphi)} \text{ (Ag)} \\
\frac{j : \varphi \quad (flex_i^0 \vee \tau_{i,j}(\varphi)) \wedge \neg(flex_i^0 \rightarrow \tau_{i,j}(\varphi)) \wedge \neg(\tau_{i,j}(\varphi) \rightarrow flex_i^0)}{i : \mathbf{comp}(j : \varphi) \cap i : flex_i^1} \text{ (Co)}
\end{array}$$

Table 3. Rules for the second proposing agent.

2. His stubbornness condition, i.e., if his CAF is $stub_B$ ($flex_B^k \leftrightarrow stub_B$) or not. Whenever B is stubborn, he performs the same counterproposal, otherwise he may relax his CAF by the (W) rule ($flex_B^k \rightarrow flex_B^{k+1}$) or change his theory by the (C) rule ($flex_B^k \vee flex_B^{k+1}$).

At the end of his evaluation, B replies to A with a counterproposal ψ . When A evaluates ψ she has to consider the relation between her CAF and ψ , her stubbornness condition ($stub_A \leftrightarrow flex_A^k$) and B 's evaluation. The evaluation of the opponent agent helps agents in choosing the new proposal. The choice of the action, weakening or changing theory, and of the next proposal depends on the agent's attitude: a *collaborative agent* chooses the proposal that improves the negotiation relation with the opponent and a *competitive agent* chooses the proposal that changes the least the relation with the opponent. For instance, if B says to A that when A proposes φ they are in essence disagreement, and B makes the proposal ψ , A will propose φ_1 or φ_2 , both inferred from φ by applying (W) or (C). When A is collaborative, she will propose φ_1 because she knows that they will be in agreement. Conversely, A will propose φ_2 , if A is competitive, because she knows that they will remain in essence disagreement. Suppose B says to A that when A proposes φ they are in relative disagreement ($\psi \rightarrow \varphi$) and B makes the proposal ψ , then A knows that they are in agreement when she proposes ψ .

To support the interaction sketched above, we define the system MND to consist of the standard introduction and elimination rules for the connectives of \mathcal{L}_i and \mathcal{L} , and of two sets of rules: one set for the second proposing agent (Table 3) and another set for the following proposing agents (Table 4). For the sake of space, we omit the assumption of non call-away conditions in negotiation rules and explain only some of the rules by example.

Assume that A begins a MN by proposing $flex_A^0$ to B . B evaluates $\tau_{B,A}(flex_A^0)$ with respect to his initial angle $flex_B^0$ and suppose B thinks that $\tau_{B,A}(flex_A^0)$ is too strict, i.e., $\tau_{B,A}(flex_A^0) \rightarrow flex_B^0$. Thus, B cannot accept $\tau_{B,A}(flex_A^0)$ and re-initiates the MN by the rule (I) and proposes $flex_B^0$ by $B : flex_B^0$. Otherwise, suppose B thinks that $\tau_{B,A}(flex_A^0)$ is entailed by his initial angle $flex_B^0$ and that $\tau_{B,A}(flex_A^0)$ is not too general, i.e., it is not entailed by $stub_B$. In this case, B knows that A cannot accept $flex_B^0$ because it is too strict with respect to her viewpoint, thus if B accepts $\tau_{B,A}(flex_A^0)$ by (Ag) and says $B : \mathbf{agree}(A : flex_A^0)$. This is the reason why there is no rule (RD) in Table 3. Consider the case in which B thinks that the proposal of A , $flex_A^0$, is consistent to his initial angle $flex_B^0$ by (Co) . B says to A that they are in the compatibility relation by $B : \mathbf{comp}(A : flex_A^0)$ and makes a new proposal $B : flex_B^1$ such that $flex_B^0 \rightarrow flex_B^1$ (rule (W)). Now A thinks that $\tau_{A,B}(flex_B^1)$ is an acceptable angle of her initial viewpoint, i.e. $flex_A^1 \leftrightarrow \tau_{A,B}(flex_B^1)$. Thus A agrees with B and says $A : \mathbf{agree}(B : flex_B^1)$ by $(Co-Ag)$. It may be the case that agents make proposals that become inconsistent with the received one. This inconsistency is tested by the opponent agent, not by the bidding one, because in MND agents choose the new proposal only with respect to their angles and not with respect to the opponent's one.

Consider now the scenario in Figure 2(b). B evaluates the proposal of A , tests the compatibility relation, and makes the counterproposal. A evaluates it and finds they are inconsistent. In situations like this, agents perform proposals that violate the MN relation among agents; we call such a proposal a *violation* and the rule causing it a *violation rule*. In Table 4, the violation rules are $(ED-AD)$ and $(ED-Co)$.

The MN develops by agents making proposals and asserting if they are in agreement or not. The entire process is controlled by a supervisor, an external viewpoint, which tests if the MN ends and if the outcome is positive or negative. Table 5 shows the transition rules for the system, which are a translation of the system transition graph in Figure 1(a). We use $j : \mathbf{na}(i : \varphi)$ to say that agent j thinks she is not in agreement with $i : \varphi$ and $*(i, j)$ to say *whatever the system state is* different from the final ones (*Agreement* and *Disagreement*), i.e., whether the system is in *Init* or *Negotiate*.⁴ The MN begins when agents make proposals in turns ($i : \varphi, j : \psi$) and they are not in agreement ($j : \mathbf{na}(i : \varphi)$) by (N) . The MN ends with a positive outcome (φ) when each agent agrees on a

⁴ An agent is *absolutely stubborn* when she only has unquestionable knowledge. If all the involved agents are absolutely stubborn then the finite state diagram is different from Figure 1(a) because the state *Negotiate* does not exist and there are only the dashed edges. However, the formalization above works as well.

proposal ($j : \mathbf{agree}(i : \varphi)$), otherwise the MN ends with a negative outcome if there are no more proposals to perform ($stub_i \leftrightarrow \varphi$ and $stub_j \leftrightarrow \psi$) and agents do not agree on a common acceptable angle ($j : \mathbf{na}(i : \varphi)$).

Example 3. Let Alice and Bob two negotiating agents discussing about the definition of the term “vehicle”. Suppose that the initial viewpoint of Alice is

$$flex_A^0 = \text{has2wheels} \wedge \text{hasSteeringWheel} \wedge \text{hasMotor}$$

and her stubbornness knowledge is as in Example 2. Suppose that Bob’s initial viewpoint is

$$flex_B^0 = \text{has2wheels} \wedge \text{hasHandleBar} \wedge \text{has2pedals}$$

and his stubbornness knowledge is

$$\begin{aligned} stub_B = & (\text{has2wheels} \vee \text{has3wheels} \vee \text{has4wheels}) \wedge \\ & (\text{hasHandleBar} \vee \text{hasSteeringWheel}) \wedge \\ & (\text{hasMotor} \vee \text{has2pedals} \vee \text{has4pedals}) \end{aligned}$$

Alice is the first bidding agent and she proposes $flex_A^0$ to Bob. Bob receives the proposal and evaluates it. Bob tests that they are in compatibility because $(flex_B^0 \vee \tau_{B,A}(flex_A^0)) \wedge \neg(flex_B^0 \rightarrow \tau_{B,A}(flex_A^0)) \wedge \neg(flex_B^0 \leftarrow \tau_{B,A}(flex_A^0))$. Bob chooses the new CAF by a weakening action (W) in

$$flex_B^1 = (\text{has2wheels} \vee \text{has4wheels}) \wedge (\text{hasHandleBar} \vee \text{hasSteeringWheel}) \wedge \text{has2pedals}$$

Bob uses the (Co) rule and sends his CAF to Alice:

$$\frac{A : flex_A^0 \quad (flex_B^1 \vee \tau_{B,A}(flex_A^0)) \wedge \neg(flex_B^1 \rightarrow \tau_{B,A}(flex_A^0)) \wedge \neg(flex_B^1 \leftarrow \tau_{B,A}(flex_A^0))}{B : \mathbf{comp}(A : flex_A^0) \cap B : flex_B^1} \quad (Co)$$

The system continues the MN by:

$$\frac{* (A, B) \quad A : flex_A^0 \quad B : \mathbf{comp}(A : flex_A^0) \quad B : flex_B^1}{Negotiate(A, B)} \quad (N)$$

Alice receives $flex_B^1$ and she has to make a weakening or a changing theory action because Bob did not say they were in agreement nor in relative disagreement. Alice performs a changing theory action by (W)-rule and her CAF is

$$flex_A^1 = \text{has2wheels} \wedge (\text{hasHandleBar} \vee \text{hasSteeringWheel}) \wedge \text{has2pedals}$$

Alice thinks they are in relative disagreement since $(flex_A^1 \rightarrow \tau_{A,B}(flex_B^1)) \wedge \neg(flex_A^1 \leftarrow \tau_{A,B}(flex_B^1))$. Alice uses the rule ($Co-RD$) and informs Bob that they are in relative disagreement:

$$\frac{B : \mathbf{comp}(A : flex_A^0) \cap B : flex_B^1 \quad (flex_A^1 \rightarrow \tau_{A,B}(flex_B^1)) \wedge \neg(flex_A^1 \leftarrow \tau_{A,B}(flex_B^1))}{A : \mathbf{relDis}(B : flex_B^1) \cap A : flex_A^1} \quad (Co-RD)$$

The system continues the MN by:

$$\frac{*(B, A) \quad B : flex_B^1 \quad A : \mathbf{relDis}(B : flex_B^1) \quad A : flex_A^1}{Negotiate(B, A)} (N)$$

Bob receives $flex_A^1$ and he accepts it because Alice said they are in relative disagreement.

$$\frac{A : \mathbf{relDis}(B : flex_B^1) \cap A : flex_A^1}{B : \mathbf{agree}(A : flex_A^1) \cap B : \tau_{B,A}(flex_A^1)} (RD-Ag)$$

The system closes the MN by:

$$\frac{*(A, B) \quad A : flex_A^1 \quad B : \mathbf{agree}(A : flex_A^1)}{Agreement(A, B)} (A)$$

with a positive outcome, $flex_A^1$.

In Figure 3 we show the message passing flow between Alice and Bob (Figure 3(a)), and the changes of their E/Y configurations (Figure 3(b)).

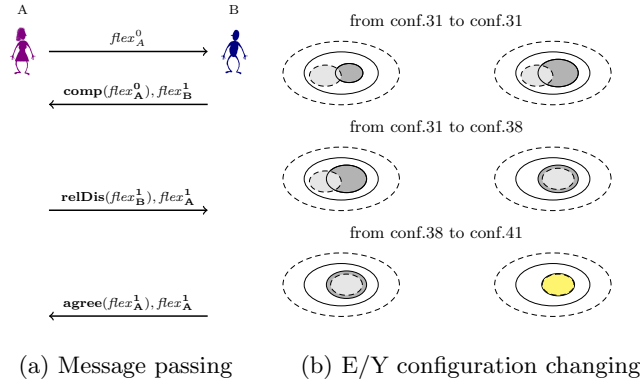


Fig. 3. The MN scenario of Example 3: the message passing flow (a) and the changes of agents' CAFs (b).

The classification of the agreement conditions provided above is complete, in the sense that there is no other possible configuration of EGG/YOLKS, as shown in [12]. Based on the completeness of that analysis, we can show the following results.

Theorem 1. *MND is consistent.*

Proof. Consider two agents represented in the *MND* system with sets \mathcal{L}_{S_1} and \mathcal{L}_{S_2} of stubbornness formulas and sets \mathcal{L}_{F_1} and \mathcal{L}_{F_2} of flexible formulas. To

prove that *MND* is consistent, we show that if a Σ_i formula ξ is inferred using the *MND* rules, or, in other terms, is deduced as a theorem in the system, then ξ represents a proposal that is acceptable by both agents. In other words, we aim at proving that when the rules yield ξ then ξ generalizes both $\mathcal{L}_{\mathcal{F}_1}$ and $\mathcal{L}_{\mathcal{F}_2}$ and is generalized by both $\mathcal{L}_{\mathcal{S}_1}$ and $\mathcal{L}_{\mathcal{S}_2}$. To prove this claim, we need specifically to show that:

- The rules for making new proposals yield a relation that is acceptable from the viewpoint of the agent who made the proposal before and infer a new proposal again still acceptable. In other terms, if an agent makes a proposal that is generalized by the set of stubbornness formulas $\mathcal{L}_{\mathcal{S}_i}$, and is a generalization of the set of flexible formulas $\mathcal{L}_{\mathcal{F}_i}$, for one agent, the rules infer a new proposal that is in the same relationships with $\mathcal{L}_{\mathcal{S}_i}$ and $\mathcal{L}_{\mathcal{F}_i}$.
- The rules for the second proposing agent infer the relation between the agents at that step of the negotiation.
- The rules for the following proposing agent do the same as the rules for the second proposing agent, taking into account that this step takes place after the step of the second proposing agent.
- The system transition rules close the negotiation only when the proposal is acceptable by both agents, namely generalizes both $\mathcal{L}_{\mathcal{F}_i}$ and is generalized by both $\mathcal{L}_{\mathcal{S}_i}$ sets.

Let us now consider a formula ξ that is acceptable by the two agents, and let us consider the rules that produce transitions in the system. In particular, if ξ is inferred by means of one of the rules *(AD)*, *(ED)*, *(I)*, *(Co)*, *(Ag)* for the second proposing agent, or by means of one of the rules given in Figure 4 for the following proposing agent, then the possible results of the step described above are given by the application of the system transition rules. Evidently, if ξ is inferred, then the rule *(D)* does not apply. If *(N)* applies, and one more inference is performed, then the rules *(W)*, *(C)*, *(S)* allow us to infer a different formula. Suppose now, by contradiction, that the new formula ξ is not acceptable by one of the agents (in the sense that either is not a generalization of her set of flexible formulas or it is not generalized by the set of stubbornness formulas. As a consequence, one agent has called herself away, as we stated above. This, however, is impossible, by construction of the rules for the second and following proposals. Conversely, if the transition rule *(D)* applies and, therefore, the agents have incompatible viewpoints, then ξ is not inferred through the system, because it is not a generalization of both flexible sets of formulas and generalizes by both stubbornness sets of formulas. Clearly, by means of the full set of rules, it is not possible to do so when the agents have compatible viewpoints. \square

Theorem 2. *MND is adequate to represent the MN of two agents.*

Proof. We consider two agents that have compatible viewpoints, namely such that there exists a possible common angle. Their stubbornness sets and their flexible sets of formulas are in one of the EGG/YOLK configurations that do not correspond to one of the call-away or absolute disagreement relations. Suppose now that the *MND* system infers a Σ_i formula ξ . Then, ξ is a common

angle. Conversely, consider two agents with incompatible viewpoints. The relation established is either call-away or absolute disagreement. The result is that no formula can be inferred through the system, which is consistent by Theorem 1. Hence, overall, the system is adequate. \square

For MN processes that are built on finite signature theories, we obtain the following decidability result:

Corollary 1. *MN is decidable for theories with finite signature under the assumption of competitive agents.*

Proof. Consider an MN between competitive agents on a language with finite signature. The number of possible proposals the agents can exchange during a negotiation process is formed by the possible formulas that can be built on the signature, which is finite. Since the rules of *MND* are finite and the new possible proposals are finite, and the number of applications of each rule is limited to the number of proposals the other negotiator can perform, then the number of steps that will be performed, in any algorithmic solution to the problem, is finite as well. \square

3 Conclusions

As we remarked, the literature has dealt with many different issues of the negotiation of meaning, but what has been only partially treated is the description of the process of reaching agreement conditions. This was the focus of this paper, whose main results can be summarized in three points:

- We defined the agreement conditions and classified the ways in which agents can be in disagreement. This refines the state-of-the-art, where the only distinguishable conditions are agreement and disagreement alone.
- We defined rules for deriving streams of dialog between two meaning negotiating agents.
- We defined a deduction system, *MND*, based upon these rules, which derives a stream of dialog that ends with an agreement (or disagreement) condition.

Although these results are only a first step, we believe that they show the usefulness and strength of our approach. Much is still to be done, in particular investigating the formal properties of *MND*, such as soundness and completeness. The proofs of consistency and adequacy do not fix the relation to a given semantics, which is needed for a proof of soundness and a proof of completeness. Usually, a deduction system can be proved sound and complete against a standard interpretation of the language, which is difficult to circumscribe in our case, because of the presence of the relations between agents to be represented. A standard definition of the semantics for the *MND* systems is therefore needed in front of any further investigation of the soundness and completeness properties.

In this paper, we assumed that agents are truthful thus they never inform the opponents about something wrongly. Fraudulent agents may try to drive

the MN in a way that is in some sense optimal for themselves. It would be interesting to study the optimality and minimality of the MN outcomes and the ways, legitimate or not, that the agents use to reach optimal outcomes.

It would also be interesting to develop a decision making algorithm for those cases in which the system results decidable, in particular for finite signatures in addition to the case of competitive agents we considered here. This would foster both the automation of the subjective decision process (i.e., the automation of the deduction system alone) and the automation of the whole process per se (i.e., the definition of a procedure to establish the agreement terminal condition). We shall also clarify how the different choices that every agent makes with respect to the sequence of proposals affect the general strategies and results of the MN process.

The investigation we carried out can also be extended by studying the ways in which agents can be limited to specific strategies in choosing the next action. Jointly with the definition of an algorithm for negotiating a common angle, this study can also enlarge the boundary of decidable cases. In particular, agents using some specific strategies can apply the rules in a finite number of steps even if the signature is infinite.

Finally, we envisage two further extensions of our approach: (i) to more than two negotiating agents, where it is well-known from game theory (e.g., [3]) that such an extension is all but trivial; (ii) to applications in information security, e.g., investigating the relationships between the MN process and the management of authorization policies in security protocols and web services.

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$$\begin{array}{c}
\frac{j : \mathbf{absDis}(i : flex_i^k) \cap j : \psi \quad \neg(stub_i \wedge \tau_{i,j}(\psi))}{i : \mathbf{absDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (AD-AD) \\
\frac{j : \mathbf{absDis}(i : flex_i^k) \cap j : \psi \quad (stub_i \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \wedge \tau_{i,j}(\psi))}{i : \mathbf{essDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (AD-ED) \\
\frac{j : \mathbf{absDis}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \rightarrow \tau_{i,j}(\varphi)) \wedge \neg(\tau_{i,j}(\varphi) \rightarrow flex_i^{k+1})}{i : \mathbf{comp}(j : \psi) \cap i : flex_i^{k+1}} \quad (AD-Co) \\
\frac{j : \mathbf{absDis}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \rightarrow \tau_{i,j}(\psi)) \wedge (\tau_{i,j}(\psi) \rightarrow flex_i^{k+1})}{i : \mathbf{relDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (AD-RD) \\
\frac{j : \mathbf{absDis}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \leftrightarrow \tau_{i,j}(\psi))}{i : \mathbf{agree}(j : \psi) \cap i : \tau_{i,j}(\psi)} \quad (AD-Ag) \\
\frac{j : \mathbf{essDis}(i : flex_i^k) \cap j : \psi \quad \neg(stub_i \wedge \tau_{i,j}(\psi))}{i : \mathbf{absDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (ED-AD) \\
\frac{j : \mathbf{essDis}(i : flex_i^k) \cap j : \psi \quad (stub_i \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \wedge \tau_{i,j}(\psi))}{i : \mathbf{essDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (ED-ED) \\
\frac{j : \mathbf{essDis}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \rightarrow \tau_{i,j}(\varphi)) \wedge \neg(\tau_{i,j}(\varphi) \rightarrow flex_i^{k+1})}{i : \mathbf{comp}(j : \psi) \cap i : flex_i^{k+1}} \quad (ED-Co) \\
\frac{j : \mathbf{essDis}(i : flex_i^k) \cap j : \psi \quad (\neg flex_i^{k+1} \vee \tau_{i,j}(\psi)) \wedge (\tau_{i,j}(\psi) \rightarrow flex_i^{k+1})}{i : \mathbf{relDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (ED-RD) \\
\frac{j : \mathbf{essDis}(i : flex_i^k) \cap j : \psi \quad (stub_i \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \vee \tau_{i,j}(\psi))}{i : \mathbf{agree}(j : \psi) \cap i : \tau_{i,j}(\psi)} \quad (ED-Ag) \\
\frac{j : \mathbf{comp}(i : flex_i^k) \cap j : \psi \quad (stub_i \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \wedge \tau_{i,j}(\psi))}{i : \mathbf{essDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (Co-ED) \\
\frac{j : \mathbf{comp}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \rightarrow \tau_{i,j}(\varphi)) \wedge \neg(\tau_{i,j}(\varphi) \rightarrow flex_i^{k+1})}{i : \mathbf{comp}(j : \psi) \cap i : flex_i^{k+1}} \quad (Co-Co) \\
\frac{j : \mathbf{comp}(i : flex_i^k) \cap j : \psi \quad (\neg flex_i^{k+1} \vee \tau_{i,j}(\psi)) \wedge (\tau_{i,j}(\psi) \rightarrow flex_i^{k+1})}{i : \mathbf{relDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (Co-RD) \\
\frac{j : \mathbf{comp}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \leftrightarrow \tau_{i,j}(\psi))}{i : \mathbf{agree}(j : \psi) \cap i : \tau_{i,j}(\psi)} \quad (Co-Ag) \\
\frac{j : \mathbf{relDis}(i : \varphi) \cap j : \psi}{i : \mathbf{agree}(j : \psi) \cap i : \tau_{i,j}(\psi)} \quad (RD-Ag)
\end{array}$$

Table 4. Rules for the following proposing agents.

$$\begin{array}{c}
\frac{* (i, j) \quad i : \varphi \quad j : \mathbf{na}(i : \varphi) \quad j : \psi \quad stub_i \leftrightarrow \varphi \quad stub_j \leftrightarrow \psi}{Disagreement(i, j)} \quad (D) \\
\frac{* (i, j) \quad i : \varphi \quad j : \mathbf{agree}(i : \varphi)}{Agreement(i, j)} \quad (A) \quad \frac{* (i, j) \quad i : \varphi \quad j : \mathbf{na}(i : \varphi) \quad j : \psi}{Negotiate(i, j)} \quad (N)
\end{array}$$

Table 5. System transition rules.